What happened at IPAM?

(one mathematician's view)

Amie Wilkinson
Mathematics, University of Chicago

Who am I?

My research lies in the area of smooth dynamical systems and is concerned with the interplay between dynamics and other structures in pure mathematics -- geometric, statistical, topological and algebraic.

One day, about 4 years ago…

Sergei Nagaitsev (Fermilab)

Friends (new and old)

We organized a workshop

Workshops

Programs > Workshops > Beam Dynamics

Beam Dynamics

JANUARY 23 - 27, 2017

OVERVIEW | SPEAKER LIST | SCHEDULE

Overview

Particle beams, from heavy ions to electrons and photons, are used to explore matters at the molecular, atomic and subatomic level, and in many industrial and medical applications. Accelerators were invented in the 1930s to provide high-energy particles to investigate the structure of the atomic nucleus. Since then, high-energy accelerators led to the discovery of the fundamental building blocks of the Universe and the exploration of the forces acting between them. From the 1970s, the field of accelerator science widened in scope from elementary particle physics to sciences exploring the structure and dynamics of organic and inorganic aggregates of atoms and molecules through the use of neutrons, synchrotron radiation, and free electron lasers. Approximately 30,000 accelerators are currently used to diagnose and treat cancer and other diseases, improve manufacturing processes, and study energy, environmental and security issues. The operation and future improvement of particle accelerators requires the solutions to challenging mathematical problems related to single particle nonlinear dynamics and collective phenomena in high intensity particle beams interacting with electromagnetic fields and plasmas. These challenges include the effects of linear and nonlinear resonances and KAM dynamics in particle accelerators, regular and chaotic effects in many body systems, collective effects, particle beam instabilities and Landau damping.

The workshop is dedicated to better understand and extend the mathematical methods available to accelerator physicists to make progress in understanding and controlling the physics and technology of these systems.

This workshop will include a poster session; a request for posters will be sent to registered participants in advance of the workshop.

IPAM was founded in 2000 by Mark Green, Tony Chan, and Eitan Tadmor as an **NSF Mathematical Sciences Institute** with a grant from the NSF Division of Mathematical Sciences. Over 2,000 visitors per year attend its workshops, long programs, student research programs, summer schools, and other programs.

Goals of workshop

For mathematicians to learn about accelerator physics, and for physicists to learn some relevant mathematical developments.

To start to develop a common language between mathematicians (many of them “pure”) and accelerator physicists.

To try to formulate problems from accelerator dynamics as mathematical problems, to interest more mathematicians.

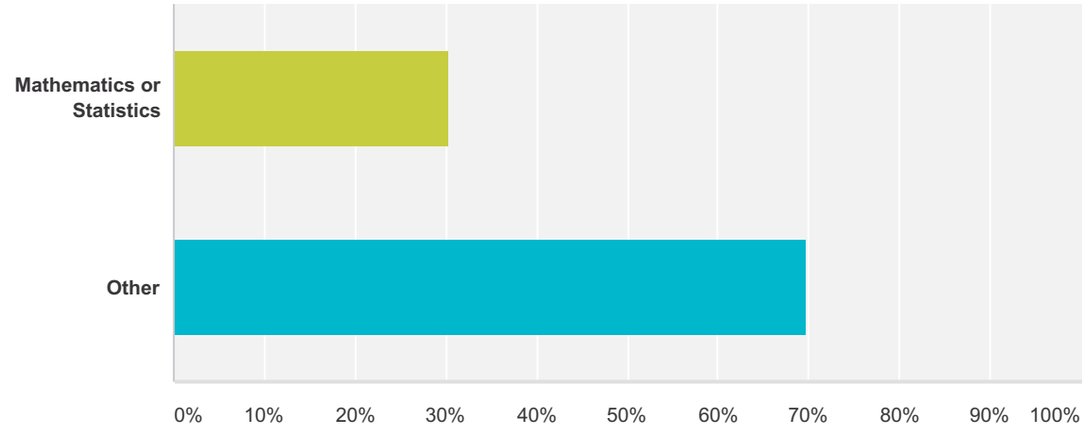
Sort out computational from theoretical problems and explore their interface.

Speakers

Enrico Allaria (Elettra Sincrotrone Trieste)
Rafael de la Llave (Georgia Institute of Technology)
Diego del-Castillo-Negrete (Oak Ridge National Laboratory)
Alex Dragt (University of Maryland)
James Ellison (University of New Mexico, Mathematics and Statistics)
Gianluca Geloni (European XFEL)
Marian Gidea (Yeshiva University)
Zhirong Huang (Stanford University)
Konstantin Khanin (University of Toronto)
Kwang-Je Kim (University of Chicago)
Ryan Lindberg (Argonne National Laboratory)
Tere Martinez-Seara (Universitat Politecnica de Catalunya)
James Meiss (University of Colorado Boulder, Mathematics)
Konstantin Mischaikow (Rutgers University New Brunswick/Piscataway)
Warren Mori (University of California, Los Angeles (UCLA),
Sergei Nagaitsev (University of Chicago)
Claudio Pellegrini (SLAC National Accelerator Laboratory)
Leonid Polterovich (Tel-Aviv University)
Sven Reiche (Paul Scherrer Institut, GFA)
David Rubin (Cornell University)
James Sethna (Cornell University)
Luis Silva (Instituto Superior Tecnico, University of Lisbon)
Gennady Stupakov (SLAC National Accelerator Laboratory)
Yine Sun (Argonne National Laboratory)

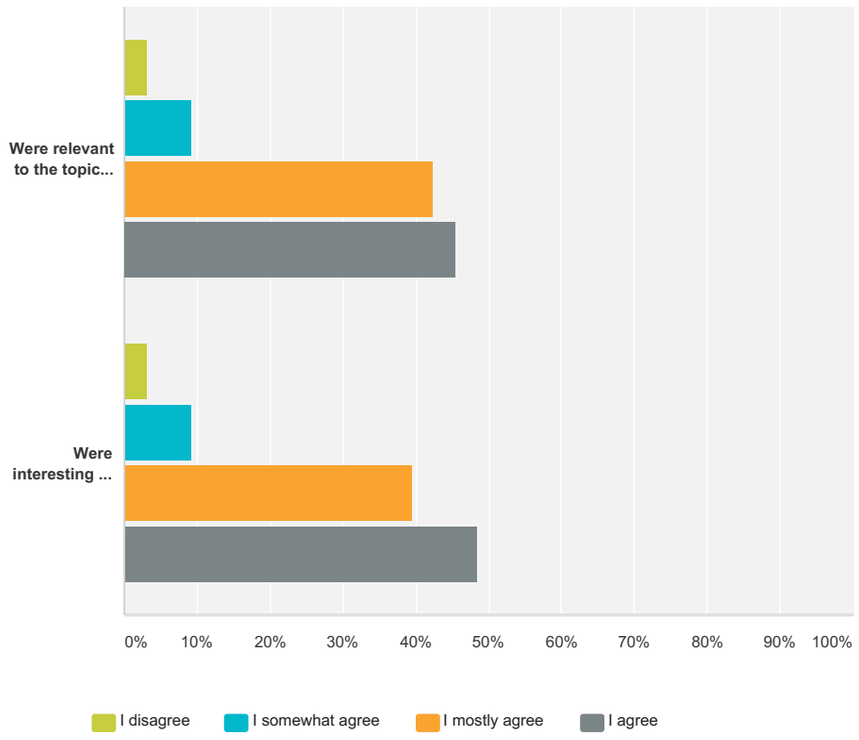
Outcomes

64 participants.



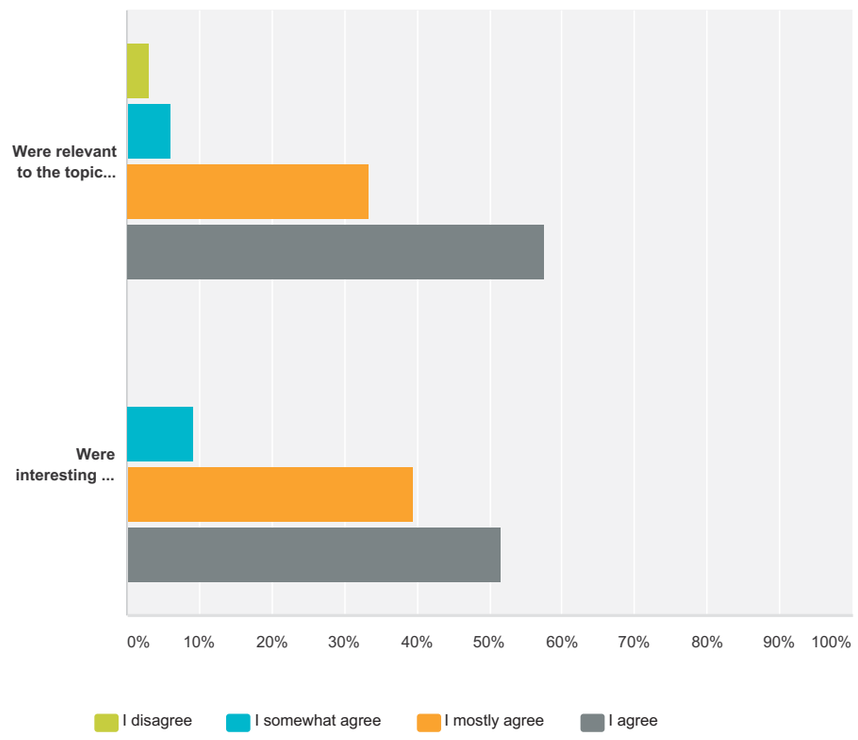
Q5 Do you agree that the lectures by mathematicians:

Answered: 33 Skipped: 0



Q6 Do you agree that that the lectures by non-mathematicians (skip if not applicable):

Answered: 33 Skipped: 0



The conference: comments from participants.

This is the first attempt of meeting particle beam dynamics with mathematician, and was successful. The experience will help to fine-tune the workshop organization in the future.

The speaker are famous experts on our fields. But the problem is they do not have enough time to reveal the beautiful and interesting details in their researches, but I understand this is impossible to overcome because this is one week workshop, not one month.

I have to say the slice of the fruits on the breakfast is too large. If they can be cut one or more times, that would be great.

Joint Physics/Mathematics workshops of this kind meet a real need for cross fertilization.

http://www.ipam.ucla.edu/programs/workshops/beam-dynamics/?tab=schedule

Broad Themes

Measurement and detection: how do we "define" aperture? How do we determine actual strength of magnets (e.g. sextupoles).

Optimization: Everyone uses genetic algorithms. Are they "all that?" What other optimization techniques better suited to the physics might be used?

Prediction and design: Is it possible to determine dependence on parameters more explicitly to avoid heavy use of Monte Carlo methods? Rings and FELs "by design"?

THE TALKS

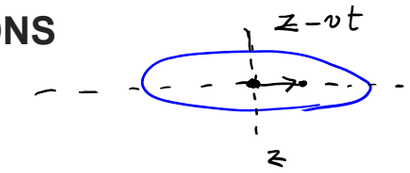
Particle accelerators in science and technology: status, future developments and challenges

Claudio Pellegrini, SLAC

Three Beam Dynamics Problems

Kwang-Je Kim (Argonne and Chicago)

VARIABLES AND EQUATIONS



- Variables:
  - "Time" : z
  - Position: $\zeta = z - v_0 t$
  - Momentum: $\Delta\beta \equiv d\zeta / dz = 1 - \beta_0 / \beta = (\Delta\gamma / \gamma)(1 / \beta\gamma^2)$
- Electron motion:

$$d\zeta / dz = \Delta\beta, \quad d\Delta\beta / dz = eE / (mc^2 \beta\gamma^3)$$

- Klimontovich density: $f(\zeta, \Delta\beta; z) = \sum_i \delta(\zeta - \zeta_i(z))\delta(\Delta\beta - \Delta\beta_i(z))$
- Continuity: $\dfrac{\partial f}{\partial z} + \Delta\beta \dfrac{\partial f}{\partial \zeta} + \dfrac{eE}{mc\beta\gamma^3} \dfrac{\partial f}{\partial \Delta\beta} = 0$
- Maxwell (Gauss-Poisson) equation for the longitudinal electric field E

$$\left.\frac{\partial E}{\partial z}\right|_t = \frac{\partial E}{\partial z} + \frac{\partial E}{\partial \zeta} = \frac{e}{\varepsilon_0 \Sigma_A} \int d\Delta\beta f$$

Small

Argonne

PERTURBATION SCHEME

- Decompose f into smooth background and the rest:
  - $f = f_0 + \hat{f}$
  - f_0: smooth background, treat as the zeroth order:

$$f_0(\Delta\beta) = n_0 g(\Delta\beta) \qquad g(\Delta\beta) = \exp(-\Delta\beta^2 / 2\sigma_{\Delta\beta}^2) / \sqrt{2\pi}\sigma_{\Delta\beta}$$

  - $\hat{f}$: high frequency part, regarded as the first order
  - Source of E is $\int d\Delta\beta \hat{f}(\Delta\beta; z)$ → thus E is of the first order
- K-M equations become linear in E and $\hat{f}$:

$$\frac{\partial \hat{f}}{\partial z} + \Delta\beta \frac{\partial \hat{f}}{\partial \zeta} + \frac{eE}{mc\beta\gamma^3} \frac{df_0}{d\Delta\beta} = 0; \quad \frac{\partial E}{\partial \zeta} = \frac{e}{\varepsilon_0 \Sigma_A} \int d\Delta\beta \hat{f}$$

- Introduce Fourier transform in ζ and Laplace transform in z:

$$\hat{f}_{\omega,k}(\Delta\beta) = \int_0^\infty dz e^{i\omega z} \int_{-\infty}^{\infty} d\zeta e^{-ik\zeta} \hat{f}(\zeta, \Delta\beta, z), \quad E_{\omega,k} = \int_0^\infty dz e^{i\omega z} \int_{-\infty}^{\infty} d\zeta e^{-ik\zeta} E(\zeta, z)$$

- K-M equations become algebraic, containing the initial conditions $\hat{f}_k(\Delta\beta; 0)$. Solve them and perform the inverse Laplace transform.
- *These steps are identical to the perturbation analysis of Vlasov equations!*

Slides 4 and 5 of his talk (25 slides)

Speakers were constantly interrupted (good?).

He only got to ask one of the three questions (bad?).

Integrable Dynamical Systems in Particle Accelerators

Sergei Nagaitsev (University of Chicago)

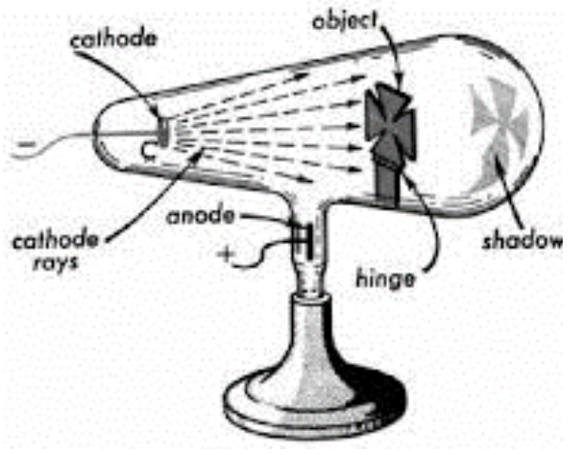
Confirmed what we mathematicians already know:

H. Poincaré is the source of everything that is good in this world.

In 1896, before the Thomson's discovery, Poincare has suggested that Birkeland's experiment can be explained by "cathode rays being charges moving in the field of a magnetic monopole"

– He wrote a brilliant paper in 1896, proving that charge motion in the field of magnetic monopole is fully integrable (but unbounded).

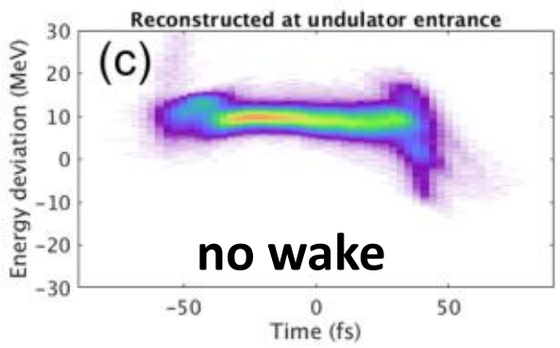
$$\mathbf{B} = \frac{k\mathbf{r}}{r^3}$$

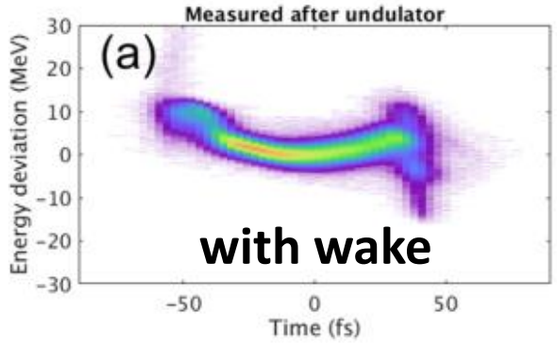


Solving High-Gain FEL systems using Van Kampen's Normal Mode Expansion

Zhirong Huang (Stanford University)

Undulator wakefield is an important source of time-dependent energy loss





Energy LOSS? Really?

The talk that didn't happen

Leonid Polterovich (Tel Aviv)

Symplectic topology and Hamiltonian dynamics

Leonid Polterovich, Tel Aviv

IPAM, January 2017

THE CONTENT

Single particle dynamics (storage rings)

Deformations of elliptic, linear symplectic maps in 4D/6D

Best methods for simulation, leveraging symplectic geometry, topology and Lie algebra methods: de la Llave, Gidea, Rubin, Dragt, Mischaikow

KAM: de la Llave, Meiss

Arnol'd Diffusion: Gidea

Both of these are perturbative. Stability and instability (e.g., Nekhorosev) beyond the perturbative regime: Khanin (Aubry-Mather), Polterovich (Hofer metric)

How to measure aperture and emittance: symplectic geometry, normal forms, Lie methods: Sethna, Rubin, Seara, Polterovich, Meiss

Single particle dynamics (storage rings)

Integrability and near-integrability

Sethna: Approximation of chaotic maps by integrable ones (away from resonance?)

Polterovich: Non-possibility in general (but unknown within realm of physically possible ones) . Gives a method for measuring distance from integrability.

Nagaitsev: Possibility to design from scratch (IOTA) nonlinear integrable systems with nice properties.

Vlasov equations warmup

Collective effects arise when bunches are dense (Debye length blah blah). Model bunches by measures.

$\varphi_t =$ flow on a space.

$m =$ probability measure on that space.

Consider the equation: $\varphi_{t*} m = m_t$

Obvious solution:

$$m_t = \frac{1}{N} \sum_{j=1}^{N} \delta_{\phi_t(x_i)}$$

In fact you can do this for any initial condition m.

Convergence of solutions? Yes, in the Wasserstein distance.

Easy!

Vlasov equations

Now add feedback mechanism (loop) so that the measure itself affects vector field and consider Hamiltonian system.

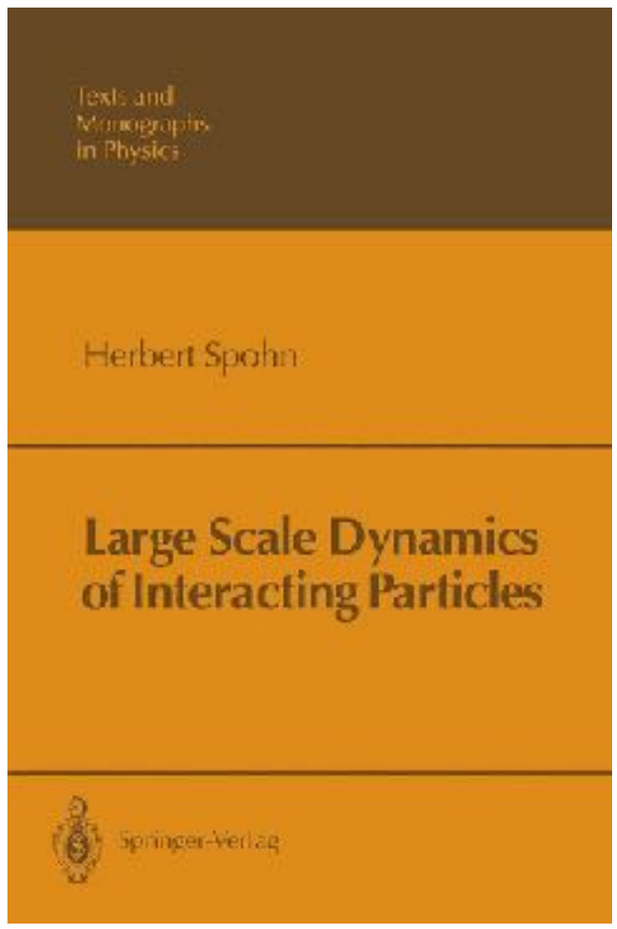
$$H(q,p,t) = \frac{1}{2}\sum_{j=1}^{N} |p_j|^2 + \phi(q,f)$$

$$m_t = f(q,p,t)|dp \wedge dq|$$

$\varphi_{t*}m = m_t$ becomes:

$$\frac{d}{dt} f(q,p,t) = 0$$

$$p\frac{\partial f}{\partial q} - \frac{d\phi}{dq}\frac{\partial f}{\partial p} + \frac{\partial f}{\partial t} = 0$$



Convergence of solutions? Yes, still.

Vlasov-Maxwell equations

PHYSICAL REVIEW VOLUME 85, NUMBER 2 JANUARY 15, 1952

A Collective Description of Electron Interactions: II. Collective *vs* Individual Particle Aspects of the Interactions

DAVID PINES
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AND

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(Received September 28, 1951)

Vlasov-Maxwell equations

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Potential is electromagnetic field. Continuity equation becomes (via Lorentz):

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial q} + e(E + p \times B) \cdot \frac{\partial f}{\partial p} = 0$$

Hamiltonian can be replaced by Maxwell equations:

$$\nabla \cdot E(q,t) = \frac{e}{\epsilon_0} \int f(q,p,t)|dp| \qquad \text{etc.}$$

$$\nabla \times B(p,t) = \frac{1}{c^2}\frac{\partial E(q,t)}{\partial t} + e\mu_0 \int f(q,p,t)\, p\, |dp|.$$

Gaussian (Maxwellian) $f_0(q,p,t) = g(p)$ is “trivial solution.”

Perturbative method for solving V-M

Perturbative method: write solution as $f = g_0 + \epsilon \hat{f}$.

Remove terms of order ϵ^2 to obtain linearized system.

Solve linearized system using Fourier methods.

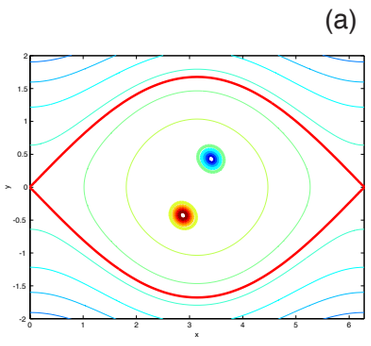
Then solution “should be” a good approximation by convergence methods (if you had solved the original equations, but you didn’t…)

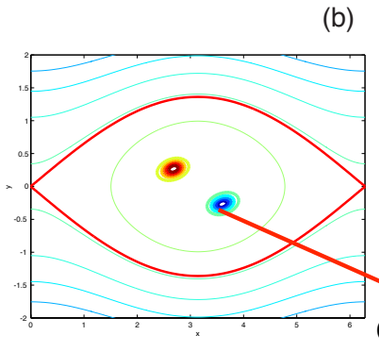
FEL, XFEL, ….. (Kim, Huang, Lindberg, Stupakov, Ellison)

V-M and magnetic confinement

Mean field models: D. del-Castillo-Negrete. Considers Vlasov equation for uncoupled harmonic oscillators driven by mean field energy ("Single Wave Model"), studying different initial conditions. Shows how fine structure (dipole dynamics) can be preserved when continuum limit is chaotic, an effect he calls "self-consistent chaos."

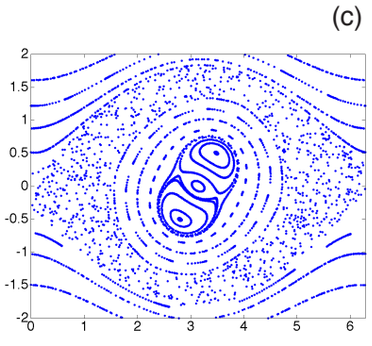
Rotating coherent dipole

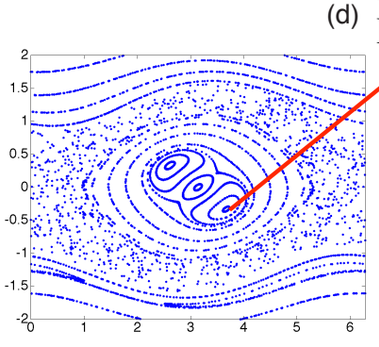




Coherence maintained by KAM surfaces

Poincare section of time periodic self-consistent mean-field





(A sample of) questions that came up.

Is there a better way to calculate dynamic aperture? Can we measure magnet parameters experimentally? (Rubin)

Is it possible to effectively implement a useful integrable nonlinear system? How to tune existing systems to get better integrability? (Nagaitsev, Sethna)

Can symplectic invariants (e.g. capacity) be used to effectively calculate quantities like emittance? (see work of B.Erdelyi). (Polterovich)

Can we use ‘shadowing’ ideas to estimate particle beam loss? (Gidea)

(A sample of) questions that came up.

Can we write algorithms that better exploit the symplectic nature of these problems? (de la Llave)

Limits of genetic algorithms — how stable is this form of optimization? (Lindberg)

How do we measure the spread of beam emittance caused by Coulomb repulsion? What is the source of the nonzero Lyapunov exponents? (Polterovich)

Can one derive large-N limit of interacting Coulomb particles, effective for both continuum and particle-level effects? (Sethna)

(A sample of) questions that came up.

Can one develop a (nonperturbative) theory of the saturated (i.e. nonlinear) regime for free electron lasers? (Huang)

Can we rigorously justify the perturbative argument? (K-J Kim)

What can we say about binary collisions of particles (coupling of short-wavelength bits)? (K-J Kim)

Can we develop a reliable, useful model of non-linear saturation in FEL? (Lindberg)